The Structural Dynamics of a Two-Body Wave Energy Converter

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Abstract

The linear equations for calculating the dynamic response of a two-body wave energy converter (WEC), which oscillates in the heave, or vertical, motion are derived. The two-body WEC comprises of a floating buoy, which is an oscillating point-absorber, and a submerged intermediate buoy, which are connected via a power take-off (PTO) system. The intermediate buoy is anchored to the sea bed using a mooring system. In this analysis, the relative response between the floating buoy and the submerged intermediate buoy is determined. This relative response defines the motion which is transmitted to the PTO system and, thus, is the convertible motion used to generate energy. A case study using this methodology is also presented.

The derivation is based on the classical structural dynamics method of using the modal equations of a multi degree of freedom system. The natural frequencies and mode shapes of the system are determined and then used to transform the coupled equations of motion into the modal equations in order to individually solve a set of uncoupled equations for the system. These equations can then be solved using a single degree of freedom numerical solution. The hydrodynamic coefficients are calculated using commercial boundary element method software.

Keywords: Modal analysis, Structural dynamics, Two-body system, Wave energy converter.

1. Introduction

Ocean wave energy is the latest natural resource to be exploited as a renewable source of energy, while also coinciding with the aim of reducing our reliance on non-renewable energy sources. The concept of harnessing ocean wave energy is by no means a new idea. However, the topic only gained international interest in the 1970's with the publication of Stephen Salter's groundbreaking paper on his Wave Energy Duck[1]. Since then thousands of patents have issued for wave energy converters (WECs), incorporating a variety of methods. However, as of yet, no 'winning' WEC design has being established.

The basic concept of a WEC is the same for each of the differing designs, regardless of the mode of motion of the wave in which it mainly utilizes. The ocean waves excite a mechanism and the motion of this mechanism is converted to pneumatic energy, which, can easily be converted to electrical energy. The system in which the mechanical energy is converted to pneumatic energy is known as the power take-off (PTO) system. The two-body system described in this paper uses the relative motion between the floating buoy and the intermediate buoy as the mechanical energy being passed to the PTO system.

In this paper, the structural dynamics of this two-body system is analysed in order to derive this motion, from which wave energy is made possible. Structural dynamics has been a major topic for civil and structural engineers for decades and many topic specific books have been written, for example by Craig[2] and Chopra[3]. The structural dynamics techniques detailed in these books will be applied to offshore structure dynamics with the ocean waves as the exciting forces. In 2007, O'Cathain et al[4] explored the time-domain multi-body modeling of marine structures and took a case study of a two-body hinged barge. Furthermore, there are a number of prototype-level two body WECs being investigated, for example the Wavebob[5] and the IPS OWEC buoy[6].

In recent years, many numerical approaches have been explored and developed in order to explore the wave-structure interaction using a numerical wave tank. For example, in 2000, Contento[7] used a 2-D numerical wave tank, which was based on the BEM technique, to simulate the nonlinear motions of arbitrary shaped bodies in order to develop improved seakeeping techniques. In 2006, Sun and Faltinsen[8] developed a 2-D numerical tank using the boundary element method in order to simulate the impact of a horizontal cylinder on the free surface. In 2007, Ning and Teng[9] used a three-dimensional higher order boundary element model to simulate a fully nonlinear irregular wave tank. In 2008, Ning et al[10] expanded this study to infinite water depth for nonlinear regular and focused waves. On the other hand, in 2011, Yan and Lui[11] developed a 3-D numerical wave tank using a high-order boundary element method in order to simulate nonlinear wave-wave and wave-body interactions. In 2012, Finnegan and Goggins[12] developed a methodology for generating linear deep water waves and performing wave-structure interaction using finite volume method software to solve the Reynolds' averaged Navier-Stokes equations.

In this paper, the hydrodynamic boundary element method software, ANSYS AQWA[13], is utilised in order to perform a hydrodynamic analysis on the floating buoy and intermediate buoy individually. The structural dynamics procedure detailed in Section 2.2 is then used to derive the relative dynamic response between the two buoys. In Section 3, a case study of a two-body wave energy converter (WEC), which oscillates in the heave, or vertical, motion, is detailed.

2. Methodology

2.1 Hydrodynamic Analysis

The total pressure distribution on a floating structure is defined by Bernoulli's equation[14]:

$$p = -\rho \frac{\partial \Phi}{\partial t} - \rho g z \tag{1}$$

where, p is pressure, ρ is the density of water, g is gravity, Φ is the velocity potential, t is time and z is the downwards distance from the SWL. The first term in Equation (1) refers to the hydrodynamic pressure distribution of the incident wave and the second term in Equation (1) refers to the hydrostatic pressure on the body in still water. Therefore, when performing the hydrodynamic analysis on a floating or fixed structure only the first term is of interest. The velocity potential, Φ, is discretised as follows:

$$\Phi = \Phi_I + \Phi_D + \Phi_R \tag{2}$$

where the incident wave velocity potential is $\Phi_I = Re[\phi_I e^{-i\omega t}]$, the diffraction velocity potential is $\Phi_D = Re[\phi_D e^{-i\omega t}]$, and the radiation potential is $\Phi_R = Re[-\sum_{k=1}^{6} i\omega u_k \phi_R e^{-i\omega t}]$. $i = \sqrt{-1}$, ω is the angular frequency, t is time, u_k is the k-component of the dynamic response of the structure and k = 1,2,3,4,5,6, corresponding to surge, sway, heave, roll, pitch and yaw. The calculation is then divided into two problems: the scattering problem, where the structure is held in a fixed position in the presence of an incident wave, to obtain the diffraction velocity potential and the radiation problem, where the structure is forced to oscillate, to obtain the radiation velocity potential.

In this paper, the numerical method used to perform the hydrodynamic analysis is based on a non-linear approximation, Stokes' 2nd order expansion[15]. Therefore the boundary conditions that need to be satisfied, in the boundary value problem, are: Laplace's equation, the kinematic free surface condition, the dynamic free surface condition, the deep water condition and the structural boundary condition, respectively[16]:

$$\nabla^2 \Phi = 0 \tag{3}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial \Phi}{\partial y}\frac{\partial \eta}{\partial y} = \frac{\partial \Phi}{\partial z}, \quad on\ z = \eta(x,y,t) \tag{4}$$

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}|\nabla \Phi|^2 + g\eta = 0, \quad on\ z = \eta(x,y,t) \tag{5}$$

$$|\nabla \Phi| \to 0, \quad as\ z \to \infty \tag{6}$$

$$\frac{\partial \phi}{\partial n} = \begin{cases} 0 & for\ scattering\ problem \\ n_j & for\ radiation\ problem \end{cases} \tag{7}$$

where, η is the vertical elevation of a point on the free surface, n_j is the unit normal in the j-direction, j = 1,2,3,4,5,6, corresponding to surge, sway, heave, roll, pitch and yaw and (x, y, z) is the Cartesian coordinate system. The j-component of the excitation force, in the frequency domain, $\hat{F}_{ext,j}$, is calculated from the scattering problem as follows:

$$\hat{F}_{ext,j} = i\rho\omega \int_{S_B} (\phi_I + \phi_D) n_j \, dS \tag{8}$$

where S_B is the wetted surface of the floating structure. The j-component of the radiation force, in the frequency domain, $\hat{F}_{R,j}$, is calculated from the radiation problem and, in turn, the hydrodynamic coefficients, the added mass and radiation damping, are determined as follows:

$$\hat{F}_{R,j} = \rho\omega^2 u_k \int_{S_B} \phi_R n_j \, dS = \omega^2 u_k a_{m,jk} + i\omega u_k v_{jk} \tag{9}$$

where a_m is the added mass and υ is the radiation damping. The dynamic response of the body can then be calculated using the response amplitude operator (RAO), which is given as:

$$\frac{u_k}{A} = \frac{\hat{F}_{ext,j}/A}{-\omega^2(m + a_{m,jk}) + i\omega v_{jk} + \tau_{jk}} \tag{10}$$

where A is the amplitude of the incident wave, m is the mass of the structure and τ is the hydrostatic stiffness of the structure.

In order to determine the various velocity potentials and calculate the hydrodynamic coefficients and forces, the boundary element method software, ANSYS AQWA[13], is used. The Green's function equation, the method for which is detailed by Garrison[17], is utilised in order to satisfy the boundary conditions in determining the velocity potentials.

2.2 Mathematical Formulation

The system being analysed in this paper is be summarised using the diagram shown in Fig 1, where, k_i is the hydrostatic stiffness of the ith buoy, k_{ms} is the mooring system stiffness, ν_i is the wave damping of the ith buoy, c_{ms} is the mooring system damping, k_{PTO} is the PTO stiffness, c_{PTO} is the PTO damping, M_i is the mass of the ith buoy, $a_{m,i}$ is the added mass of the ith buoy and F_i is the ith wave excitation force. Throughout this paper, the subscript '*1*' is denoted to the floating buoy and the subscript '*2*' is denoted to the intermediate buoy.

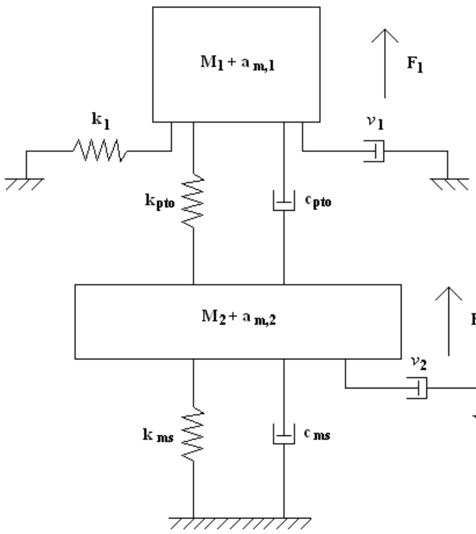


Figure 1: Free body diagram of the two-body system

The equation of motion for the system is given in matrix form, as follows:

$$[\mathrm{M}]\{\ddot{\mathrm{u}}\} + [\mathrm{C}]\{\dot{\mathrm{u}}\} + [\mathrm{K}]\{\mathrm{u}\} = \{\mathrm{F_{ext}}\} \qquad (11)$$

where, the mass matrix:

$$[\mathrm{M}] = \begin{bmatrix} \mathrm{M_1} + \mathrm{a_{m,1}} & 0 \\ 0 & \mathrm{M_2} + \mathrm{a_{m,2}} \end{bmatrix} \qquad (12)$$

The stiffness matrix, *[K]*, terms are calculated by inducing a unit displacement individually at each of the degrees of freedom. Thus, when a unit displacement of $u_1 = 1$ and $u_2 = 0$, is induced:

$$\mathrm{k_{11}} = \mathrm{k_{PTO}} + \mathrm{k_1} \qquad (13)$$

And when a unit displacement of $u_1 = 0$ and $u_2 = 1$, is induced:

$$\begin{gathered} \mathrm{k_{12}} = \mathrm{k_{21}} = -\mathrm{k_{PTO}} \\ \mathrm{k_{22}} = \mathrm{k_{PTO}} + \mathrm{k_{ms}} \end{gathered} \qquad (14)$$

Therefore, the stiffness matrix is given as:

$$[\mathrm{K}] = \begin{bmatrix} \mathrm{k_{PTO}} + \mathrm{k_1} & -\mathrm{k_{PTO}} \\ -\mathrm{k_{PTO}} & \mathrm{k_{PTO}} + \mathrm{k_{ms}} \end{bmatrix} \qquad (15)$$

Taking a similar procedure for the PTO damping and mooring system damping and, also, allowing for the wave radiation damping interaction observed by Falnes[18], the damping matrix for the system is written as:

$$[\mathrm{C}] = \begin{bmatrix} \mathrm{c_{PTO}} + \nu_1 & -\sqrt{\nu_1\nu_2} - \mathrm{c_{PTO}} \\ -\sqrt{\nu_1\nu_2} - \mathrm{c_{PTO}} & \nu_2 + \mathrm{c_{PTO}} + \mathrm{c_{ms}} \end{bmatrix} \qquad (16)$$

The natural frequencies, ω_n, of the system are then calculated. These may be calculated using the assumption that the system is undamped. This is a valid assumption as the damped natural frequency, $\omega_d = \omega_n\sqrt{1-\zeta^2}$, where the damping ratio, ζ, is small and, therefore, $\omega_d \approx \omega_n$. Also, as it is the frequency of the structure that is being calculated, the magnitude or frequency of the applied force does not affect the result. Thus, we may assume the system to be a 'free' system for this stage of the analysis. Therefore, when calculating the natural frequencies of the system, the equation of motion, in Eqn. (11) is reduced to:

$$[\mathrm{M}]\{\ddot{\mathrm{u}}\} + [\mathrm{K}]\{\mathrm{u}\} = \{0\} \qquad (17)$$

A trial solution, where the motion of the system is assumed to be harmonic motion, is now introduced and is given as:

$$\{\mathrm{u}\} = \{\phi\}\sin(\omega t + \theta) \qquad (18)$$

Subbing into Eqn. (21), gives:

$$\begin{gathered} -\omega^2[\mathrm{M}]\{\phi\}\sin(\omega t + \theta) \\ + [\mathrm{K}]\{\phi\}\sin(\omega t \\ + \theta) = \{0\} \\ \Rightarrow \big[[\mathrm{K}] - \omega^2[\mathrm{M}]\big]\{\phi\} = \{0\} \end{gathered} \qquad (19)$$

In order to determine a non-trivial solution, $\{\varphi\} \neq 0$, and, therefore:

$$\det\big[[\mathrm{K}] - \omega^2[\mathrm{M}]\big] = 0 \qquad (20)$$

where, *det* is the determinate of the matrix. The only unknown value in this equation is the natural frequencies, ω_n, which are then calculated. The modal matrix can now be calculated using the natural frequencies. The fundamental harmonic mode, φ_1, is derived from the first natural frequency, ω_1, by subbing into Eqn. (9) and then normalising and solving for φ_{11} and φ_{21}. The second harmonic mode, φ_2, is calculated in a similar way. The second natural frequency, ω_2, is subbed into Eqn. (9) and then normalising and solving for φ_{12} and φ_{22}. The two natural modes are the two columns of the modal matrix as follows:

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \qquad (21)$$

Where φ_{ij} is the mode shape coordinate representing the position of the *ith* buoy and *jth* mode.

Since the system is assumed to be linear, or the elements of the stiffness matrix remain constant throughout the analysis, a convenient method in order to return a solution is Modal Analysis[2, 3]. The original equation of motion, in Eqn. (11) is now reintroduced and, since it is a coupled equation it is necessary to transform it into a set of uncoupled equations, or the modal equations. This transformation is performed using the modal matrix, as follows:

$$\Phi^T[M]\Phi\{\ddot{u}\} + \Phi^T[C]\Phi\{\dot{u}\} + \Phi^T[K]\Phi\{u\} = \Phi^T\{F_{ext}\}$$

$$\Rightarrow \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{q}(t)_1 \\ \ddot{q}(t)_2 \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{q}(t)_1 \\ \dot{q}(t)_2 \end{Bmatrix} + \begin{bmatrix} k'_1 & 0 \\ 0 & k'_2 \end{bmatrix}\begin{Bmatrix} q(t)_1 \\ q(t)_2 \end{Bmatrix} = \begin{Bmatrix} f(t)_1 \\ f(t)_2 \end{Bmatrix} \quad (22)$$

where m_j is the modal mass in the *jth* mode, c_j is the modal classical damping in the *jth* mode, k'_j is the modal stiffness in the *jth* mode, $f(t)_j$ is the modal force in the *jth* mode and $q(t)_j$ is the modal displacement, as a function of time, in the *jth* mode.

Therefore, each uncoupled equation is analogous to a different single degree of freedom system and can be solved in the same way as using a classic single degree of freedom solution, for example the Newmark, or average acceleration, method[19]. The displacements $q_1(t)$ and $q_2(t)$ of these systems identify the contribution made by the natural modes, φ_1 and φ_2, to the actual displaced configuration, $\{u(t)\}$, of the structure at time t. However, the methodology can also be used to calculate the frequency-domain response using the response amplitude operator (RAO), given in Eqn. (10), to solve each uncoupled equation.

2.3 Mooring system design

For the purpose of this paper, the type of moorings being used is catenary lines. The inelastic catenary equations[20] are used to calculate the vertical restraint imposed by the moorings. These are more applicable to steel wire and chain and, therefore, three 100mm chain mooring lines are used. These are distributed at intervals of equal angle (120°) from the intermediate buoy. For the purpose of this analysishe length of each mooring line is set to 200m and is anchored at an average distance of 170m, horizontally, from the WEC. From this information, the restoring force of the mooring system can be calculated, which is shown in Fig. 2.

However, since the system described in this paper is linear, it is necessary to provide a spring estimate for the restoring force. This was found to be 19.24 kN/m and a comparison between the calculated and spring estimate can be seen in Fig. 2. From Fig. 2, it is clear to see that, for a maximum vertical displacement of ±5, this is a good estimate.

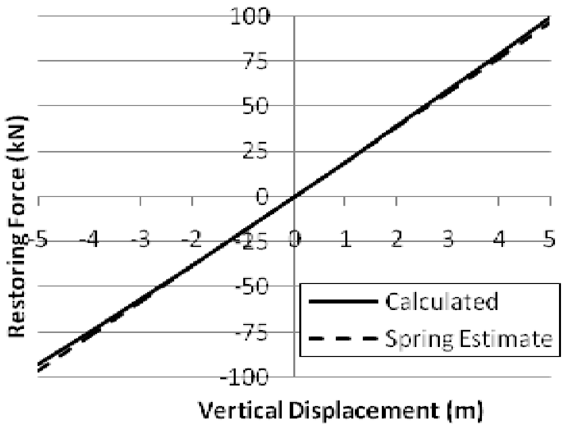
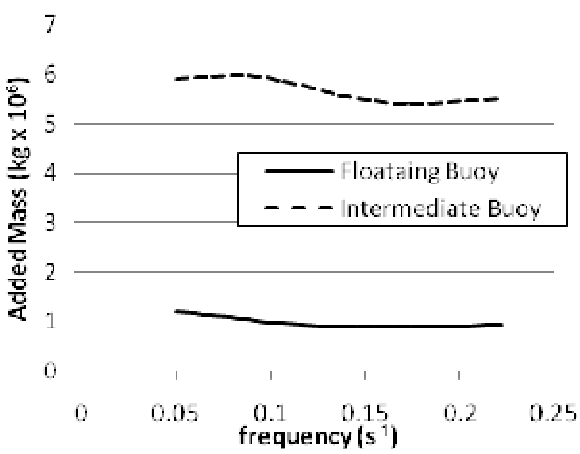


Figure 2: Restoring force of the mooring lines on the WEC

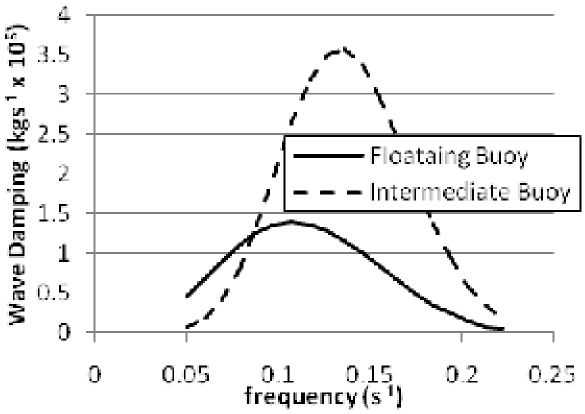
3. Case Study

In this section, the methodology described in Section 2 will be applied to a case study of a two-body WEC with a heaving vertical cylinder floating buoy, with a radius of 8m and draft of 10m, and a vertical cylinder intermediate buoy, with a radius of 12m and draft of 4m at 19m below the still water level, which is anchored to the sea bed using a catenary mooring system. The WEC is restricted to operating only in the heave, or vertical, motion for the purpose of the analysis. Thus, the system has two degrees of freedom. The PTO sytem is modelled using a spring and damper system with a damping coefficient of 100000 kg/s and an associated stiffness coefficient of 150 kN/m.

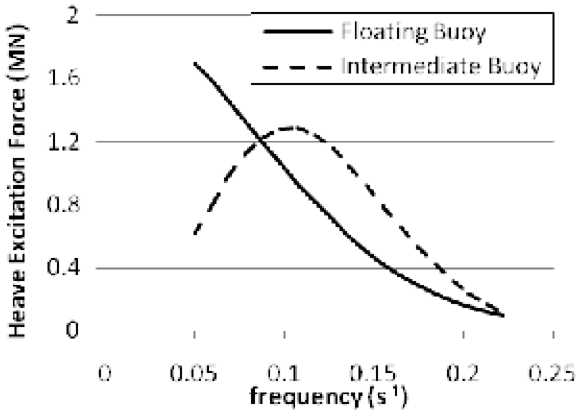
The boundary element method software, ANSYS AQWA[13], is used to perform a hydrodynamic analysis on the floating and intermediate buoys. A graphical representation of this analysis is shown in Fig. 3, detailing the added mass, wave radiation damping and the heave excitation forces on both buoys. The mooring system in the case study is assumed a linear system and is calculated using the procedure described in Section 2.3 and the spring estimate shown in Fig. 2.



(a)



(b)



(c)

Figure 3: The hydrodynamic analysis for the floating buoy and intermediate buoy, detailing (a): the added mass (b): the wave radiation damping (c): the heave excitation force

The hydrodynamic analysis is then used along with the procedure detailed in Section 2.2 to calculate the relative normalised dynamic response between the floating buoy and the intermediate buoy using the response amplitude operator. This is shown graphically in Fig. 4, in the frequency domain. Furthermore, the actual normalised dynamic response of each of the two buoys is displayed.

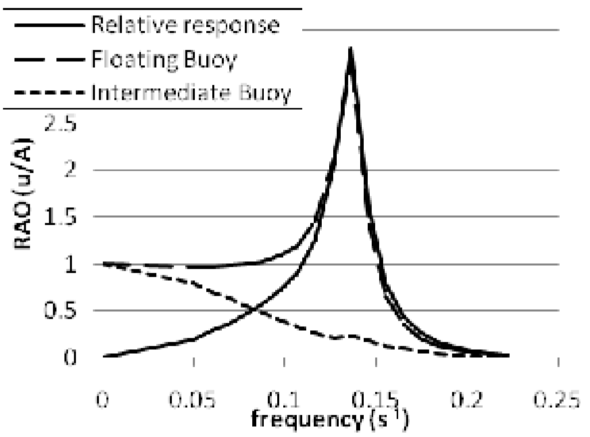


Figure 4: The response amplitude operator for each of the buoys and the relative response

4. Discussion and conclusions

From Fig. 4, it can be seen that at frequencies lower than the natural frequency of the system, the intermediate has a largely significant dynamic response, which is in phase with the floating buoy, and, therefore, the relative dynamic response is much less than that of the actual dynamic response of the floating buoy. However, at the natural frequency or at larger frequencies, the relative dynamic response is very close to that of the actual dynamic response of the floating buoy. Therefore, the two-body WEC would perform best at or greater than the natural frequency of the system.

In this paper, the linear equations for calculating the dynamic response of a two-body WEC, which oscillates in the heave, or vertical, motion are derived. The equations are derived using classical structural dynamics techniques, i.e. modal analysis, in order to solve the complex problem using a single degree of freedom method, for example the response amplitude operator in the frequency-domain or the Newmark method in the time-domain.

The two-body WEC comprises of a floating buoy, which is an oscillating point-absorber, and a submerged intermediate buoy, which are connected via a PTO system. The PTO system is modelled as a spring and damper system in this paper. The intermediate buoy is anchored to the sea bed using a mooring system, which is three catenary lines, and is modelled using a spring derived from the inelastic catenary equations.

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References

[1] S.H. Salter (1974) Wave power. *Nature*. 249(5459): p. 720-724.

[2] R.R. Craig (1981) *Structural dynamics : an introduction to computer methods*. New York: Wiley. xv, 527 p.

[3] A.K. Chopra (1995) *Dynamics of Structures: theory and applications to earthquake engineering*. Prentice-Hall international series in civil engineering and engineering mechanics. Englewood Cliffs, N.J.: Prentice-Hall.

[4] M. O'Cathain, B.J. Leira, J.V. Ringwood, *Modelling of Multibody Marine Systems with Application to Wave-Energy Devices*, in *7th European Wave and Tidal Energy Conference*. 2007: Porto, Portugal.

[5] Wavebob. *Wavebob*. 2010 [cited 2010 10-09-2010]; Available from: www.wavebob.ie.

[6] IPS. *IPS OWEC buoy*. 2012; Available from: http://www.ips-ab.com/.

[7] G. Contento (2000) Numerical wave tank computations of nonlinear motions of two-dimensional arbitrarily shaped free floating bodies. *Ocean Engineering*. 27(5): p. 531-556.

[8] H. Sun, O.M. Faltinsen (2006) Water impact of horizontal circular cylinders and cylindrical shells. *Applied Ocean Research*. 28(5): p. 299-311.

[9] D.Z. Ning, B. Teng (2007) Numerical simulation of fully nonlinear irregular wave tank in three dimension. *International Journal for Numerical Methods in Fluids*. 53: p. 1847-1862.

[10] D.Z. Ning, B. Teng, R. Eatock Taylor, J. Zang (2008) Numerical simulation of non-linear regular and focused waves in an infinite water-depth. *Ocean Engineering*. 35(8-9): p. 887-899.

[11] H. Yan, Y. Liu (2011) An efficient high-order boundary element method for nonlinear wave-wave and wave-body interactions. *Journal of Computational Physics*. 230(2): p. 402-424.

[12] W. Finnegan, J. Goggins (2012) Numerical simulation of linear water waves and wave-structure interaction. *Ocean Engineering*. 43(0): p. 23-31.

[13] ANSYS-Inc., *ANSYS AQWA, Release 13*. 2010.

[14] M.E. McCormick (1973) *Ocean Engineering Wave Mechanics*. New York: John Wiley & Sons Inc.

[15] Coastal Engineering Research Center (1977) *Shore protection manual / U.S. Army Coastal Engineering Research Center*. Fort Belvoir, Va. Washington: Supt. of Docs., U.S. Govt. Print. Off.

[16] C.M. Linton, P. McIver (2001) *Handbook of Mathematical Techniques for Wave/Structure Interactions*. Boca Raton, FL: Chapman & Hall/CRC.

[17] C.J. Garrison, *Hydrodynamic loading of large offshore structures: three-dimensional source distribution methods citation*, in *Numerical methods in offshore engineering*. 1978, Wiley.

[18] J. Falnes (2002) *Ocean Waves and Oscillating Systems*: Cambridge university Press.

[19] N.M. Newmark (1959) A method of computation for structural dynamics. *Journal of Engineering Mechanics, ASCE*. 85: p. 28.

[20] L. Johanning, G.H. Smith, J. Wolfram (2006) Mooring design approach for wave energy converters. *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment*. 220(4): p. 159-174.