Modelling high axial induction flows in tidal stream turbines with a corrected blade element model

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Abstract

The Blade Element Momentum Theory approach to turbine design is a numerically fast tool for design iteration of wind and tidal stream turbines (TST). The method works well around the optimum design point, but theoretical predictions diverge from experiments under a number of conditions. High induction, where the axial flow towards the turbine is significantly slowed, results in physically meaningless results with standard BEMT. The Buhl correction allows the axial induction factor, a, to be above the theoretical upper limit of 0.5 and to arrive at results that more closely resemble experimental conditions. The correction has been applied to wind turbines operating in air, and this is implemented here for a TST operating in water. The combined effect of tip and hub loss correction and high induction correction are compared to alternative models for the same rotor. The implementation is numerically robust across all tip speed ratios and avoids the local minima in the solution space found by traditional BEMT implementation.

Results for a number of rotor configurations are presented together with a parametric study of the axial induction factor across the whole rotor diameter and all operating speeds. The conclusions from these results are that the axial load on the rotor is increased by an amount that is not insignificant in design terms, and hence the thrust load on the foundation system of a tidal turbine will also increase above the standard BEMT predictions.

Keywords: Blade Element, ocean current, turbine, high induction

1 Introduction

The development of turbine devices to capture renewable energy from flowing water over recent years has been a motivation to create new numerical models of these devices in water. The models are often based on experience from the wind turbine industry and range from the very simplistic to fully featured transient computational fluid dynamics with fluid structure coupling. The experience of device designers is that a model is required with reasonable accuracy at low computational cost in order to analyse all of the load cases experienced by such a device over the potential lifetime of the device. This therefore limits the application of expensive models based on 3D meshes to a few select situations.

Blade element momentum theory (BEMT) is a computationally efficient method of determining the performance and loadings on a tidal stream turbine (TST). The authors have previously shown[1] that the method is sufficiently accurate for use as a design tool for tidal rotors. Efficiency improvements can be obtained by the addition of tip loss and hub loss corrections which can provide results to a level of accuracy comparable with other more computationally expensive methods [1].

It is known that blade element theory can experience certain challenges in accurately predicting rotor performance. The main causes of these problems are fundamentally due to the fact that this is a 1D model, admittedly with non-linear effects. For example, accuracy is compromised by the lack of proper modelling of three dimensional effects such as stall delay and the dependence on 2D foil data for lift and drag coefficients. There are a number of approaches to compensate for or avoid these problems however and as the computational demand is two or three orders of magnitude lower than alternatives using CFD, the

BEMT approach makes it desirable to use for a turbine rotor design and development modelling system.

Together with corrections for non-linear effects at the tip and hub, this paper focuses on a modification of the BEMT model to include a turbulent wake state correction. At high tip speed ratios (TSR) the axial induction factor can approach or exceed the theoretical upper limit of 0.5, this creates physical inconsistencies in the model as the theory implies that the flow is reversed downstream of the turbine. For this case, the momentum equations can no longer accurately describe the behaviour of the turbine. In reality, the flow downstream slows and fluid is drawn in from outside of the rotating wake, increasing turbulence. This condition is known as the turbulent wake state [2].

2 Momentum Theory

This model calculates the energy absorption of a wind or tidal turbine. The rotor is modelled as a frictionless, irrotational, permeable 'Actuator Disc'. A control volume, bounded by a stream tube, flows through the actuator disc. The actuator disc removes energy from the stream-tube by a drag force that produces a pressure drop in the fluid just downstream of the disc. Both upstream and downstream surfaces of the stream-tube are assumed to be at ambient static pressure and therefore conservation of energy requires the flow speed to reduce downstream.

Two non-dimensional induction factors are defined that characterise the amount of axial and rotational change to the momentum. The axial induction factor, a, is the ratio between the far upstream stream velocity U and the flow at the disk u. The angular (or tangential) induction factor, b, is the ratio of the angular velocity of the swirl just downstream of the rotor ω to the rotational speed of the rotor Ω

$$a = \frac{U-u}{U} \qquad b = \frac{\omega}{2\Omega}$$

The axial force applied to the actuator disc can be calculated from the rate of change of momentum of the fluid. If the stream tube is divided into annular sections with local radius r and thickness dr, the area of the stream tube annulus is $2\pi r dr$. Therefore the axial thrust is given by dF_{A1} and the torque produced on the rotor annulus is dT_1,

$$dF_{A1} = 2\pi r \frac{1}{2}\rho U^2 4a(1-a)dr$$

$$dT_1 = 4b(1-a)\rho U \Omega r^2 \pi r dr$$

3 Blade Element Theory

The discrete rotor annulus of momentum theory has an analogous 2D aerofoil section located at the same radius. Blade element theory calculates the loadings on this aerofoil based on lift and drag. dL is the element lift force and dD is the element drag force. ϕ is the inclination of the resultant flow, V, to the horizontal axis. This resultant flow is the combined vector of the axial in-flow and the rotational induced swirl at the rotor plane as well as its own rotational linear speed. α is the angle of attack of the turbine blade from the resultant flow and θ is the combined pitch and twist of the blade. The axial thrust of the individual blade element and the torque produced can be found by resolving the lift and drag forces. Therefore, expressions for dF_{A2} and dT_2 can now be obtained in terms of lift and drag coefficients, C_L and C_D. There are N blades each of chord length c.

$$dF_{A2} = N\frac{1}{2}\rho V^2 c(C_L\cos\phi + C_D\sin\phi)dr$$

$$dT_2 = N\frac{1}{2}\rho V^2 cr(C_L\sin\phi - C_D\cos\phi)dr$$

4 Corrections for Tip loss and hub loss

Blade elements are modelled individually and by implication it is assumed that there is no flow along the span of the blade. However, the pressure differential between the suction and pressure sides of the blade creates a situation where fluid will tend to flow around the tip from the pressure side to the suction side, and by implication, there is flow along the span. This flow reduces aerodynamic efficiency near the tip, reducing lift and therefore torque force and ultimately power production near the blade tip [3]. The high density of water and the larger resulting load concentration on the blade means that tidal stream turbine (TST) blades will tend to be relatively shorter than wind turbine blades, hence less closely resembling the infinite span length assumed in aerofoil theory. Therefore, tip loss will be even more significant for TSTs than for wind turbines. Glauert [4] proposed a tip loss factor F_{Tip} that is straightforward to implement into the momentum side of the BEMT equations.

$$F_{Tip} = \frac{2}{\pi}\cos^{-1}[\exp(-\left\{\frac{(N/2)[1-(r/R)]}{(r/R)\sin\phi}\right\}]$$

The hub loss equation is again taken from standard practice in wind BEMT [2] and is applied to the momentum side of the equations. The correction works inwards from the hub to model a vortex shed near the hub.

$$F_{Hub} = \frac{2}{\pi}\cos^{-1} e^{-f}$$

where

$$f = \frac{N}{2}\frac{r - R_{hub}}{r\sin\phi}$$

These two loss factors are multiplied together to obtain the total correction F and scale the momentum equations as follows:

$$dT_1 = 4Fb(1-a)\rho U\pi r^3\Omega dr$$
$$dF_{A1} = 4F\rho U^2 a(1-a)\pi r dr$$

5 High induction correction

One correction that allows for high induction is the Glauert thrust coefficient correction [4]. The model is an empirical correction of the axial thrust coefficient and is based on a.

$$a = \frac{1}{F}\left[0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_{FA})}\right]$$

This equation is employed when a exceeds *0.4* or when C_{FA} exceeds *0.96F*.

An alternative is attributed to Spera and is discussed in Shen *et al.* [5] where the critical induction factor a_c is given values between 0.2 to 0.3.

$$C_{FA} = \begin{Bmatrix} 4a(1-a)F & a \le a_c \\ 4\left(a_c^2 + (1-2a_c)a\right)F & a > a_c \end{Bmatrix}$$

$$a = \frac{1}{2}\left[2 + K(1-2a_c) - \sqrt{\left(K(1-2a_c)+2\right)^2 + 4\left(Ka_c^2 - 1\right)}\right]$$

$$K = \frac{4F\sin^2\phi}{\sigma C_L\cos\phi + C_D\sin\phi}$$

This correction system gives a linear variation of C_{FA} with a, which may over simplify the high induction relationship but lends itself to easier calibration with experimental results. This function with critical factor $a_c = 0.2$ is plotted on Fig. 1 where it can be seen that Spera's correction crosses the BEMT induction/ C_{FA} curve at the critical induction factor point and the gradient is well matched at this point. In the authors opinion this critical value is rather low to use as the transition point for changing from BEMT to an empirical high induction model however. This is especially true when the system is employed locally to blade elements rather than for the entire rotor plane where average induction factors will be lower than those values at the blade tips.

The Glauert formula was originally developed to correct the thrust coefficient for an entire rotor but has since been applied to individual blade elements. Buhl [6] explains that, near the tip, axial induction factors will tend to be high and so tip loss and axial induction correction are dependent on one another. The Glauert correction in combination with a tip loss introduces a numerical inconsistency as the correction and traditional BEMT are discontinuous at the transition stage. This discontinuity can lead to errors in the solution of the objective functions and does not match the physical behaviour of a rotor system at the transition between turbulent and non-turbulent wake. Buhl [6] introduced a modification to Glauert's theory that incorporated tip and hub losses and addressed the numerical inconsistency in Glauert's model.

$$C_{FA} = \frac{8}{9} + \left(4F - \frac{40}{9}\right)a + \left(\frac{50}{9} - 4F\right)a^2$$

Buhl simply fits a curve to the data with two aims; to match the value and slope of the BEMT curve at the crossover point (a =*0.4*) and to give the flat plate drag result, C_{FA} =*2* when a =*1*. By imposing these mathematical limits, Buhl's values will always coincide with the BEMT values at the transition point as shown in Fig. 1.

Due to this smooth transition and a reasonably high crossing between BEMT and empirical correction, the Buhl correction was selected for use in the present model. At the upper limit of the axial induction factor where a =*1* the correction gives C_{FA} =*2*, this condition is analogous with a flat plate where all flow is stopped at the rotor plane, this therefore gives some degree of confidence at the limits of the model. It is clear that these empirical corrections should be validated against tidal turbine data when this information becomes available [7].

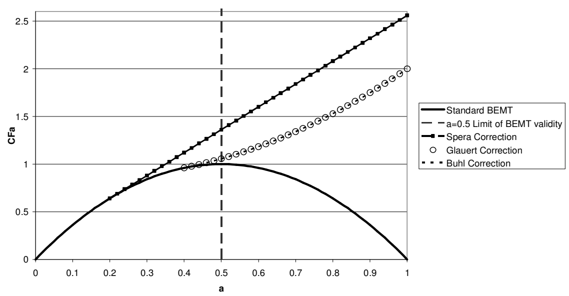

Figure 1: Axial Force Coefficient against a for standard BEMT, Spera, Glauert and Buhl corrections

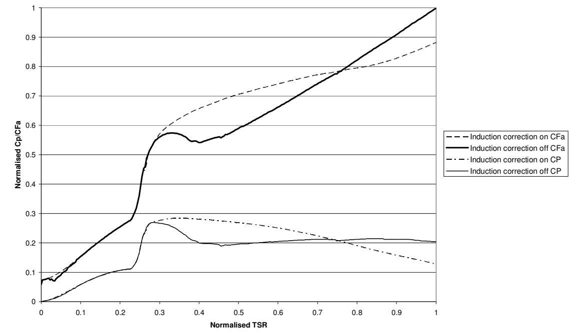

Figure 4: Rotor performance and axial thrust curves for a rotor with a large number of high induction values (blade pitched by 10° towards rotor plane). Results with and without Buhl's correction displayed.

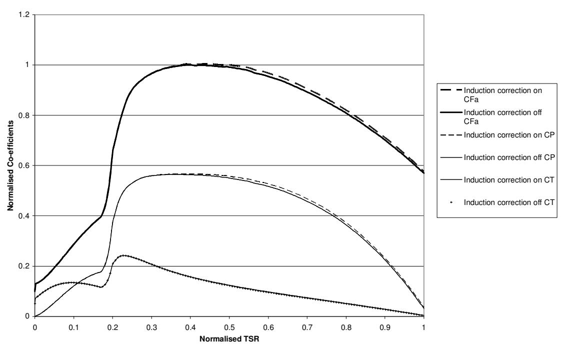

Figure 2: Rotor performance, torque and axial thrust curves for a rotor with a small number of high induction values (blade pitched by 5° towards rotor plane). Results with and without Buhl's correction displayed.

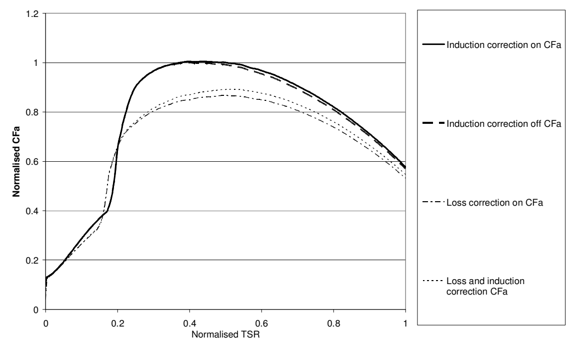

Figure 5: Axial force coefficient with combinations of tip loss, hub loss and high induction corrections (blade pitched by 10° towards rotor plane).

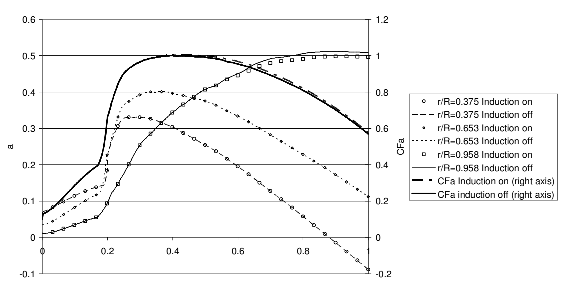

Figure 3: Variation of axial induction factor, *a,* with TSR and radius (blade pitched by 5° towards rotor plane). Results with and without Buhl's correction.

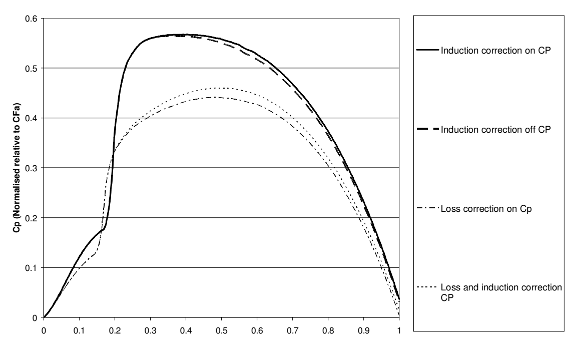

Figure 6: Power coefficient with combinations of tip loss, hub loss and high induction corrections (blade pitched by 10° towards rotor plane).

6 Results

Fig. 2 shows the results for the rotor blade profile used in the previous tip loss study [1] with the blades pitched by an extra 5 degrees towards the rotor plane. In this case, high induction values are encountered but these values do not become excessive. It can be seen that, at low TSR, there is no disagreement between the results with and without induction correction. This is as expected, in this region the axial induction factor is below 0.4 and so traditional BEMT is used in both models. With increasing TSR, high induction factors are encountered near the tip and the operation of the correction factor is evident in the axial force coefficient curve and to a lesser extent in the power coefficient curve. The code has been allowed to exceed the theoretical limit of =0.5 for the cases with and without high induction correction. Fig. 3 shows that a mathematical solution continues in excess of this physical constraint, this explains why there is only a small disagreement between the results in fig. 2.

Fig. 4 demonstrates a more extreme case where the blade is pitched by 10 degrees towards the rotor plane. This gives a far greater proportion of high induction elements and at values of normalised TSR>0.25, the traditional BEMT model struggles to find a valid solution and the majority of elements fail to converge on a solution. Using Buhl's correction, the system is able to function and produces converged solutions for axial induction factors in excess of 0.5 (the BEMT limit).

Because tip (and hub) losses lead to high local induction factors, it is important that tip, hub and high induction correction factors function in combination. It has been shown [1] that the tip and hub loss models function well in combination so it only remains to test these in combination with the high induction correction. In figs. 5 and 6, the high induction correction can be seen to have a more marked effect on the curve with losses turned on than with the losses neglected. This is because the tip and hub losses lead to higher induction factors at the tip and root. Combination of tip, hub and axial induction causes little problem using the implementation presented here. The objective function becomes more complex and so processing time is increased slightly but the solver is still able to solve the equations reliably.

7 Conclusions

In this paper tip and hub loss corrections were presented and can improve accuracy. There is only a small computational cost of employing these corrections and so inclusion proves to be beneficial. The Prandtl based corrections were selected as they do not rely on empirical data. In addition a high induction correction proposed by Buhl was implemented and results are discussed. A smooth and therefore reliable transition between the BEMT equations and this correction has been demonstrated. The uncertainty of applicability of the empirical data this correction is based on was discussed and a need for future investigation highlighted.

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