Effect of wave direction relative to wind on the motions of offshore floating wind turbine systems

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Abstract

A coupled dynamic analysis of a floating wind turbine system has been performed to investigate effect of wave direction relative to wind on the system. Hydrodynamic loads are calculated by linear frequency domain approach and aerodynamic effect is taken into account by increasing hydrodynamic damping and restoring matrices with aerodynamic damping and gyroscopic stiffness.

A modal analysis of the system was made to explain the calculated motions. It brings out the natural frequencies, natural modes and modal damping coefficients of the system. Excitation of natural modes, by waves explains the motion observed in the response amplitude operators, and the effect of wave direction relative to wind. This modal analysis helps to better understand the behaviour of floating wind turbine systems.

Keywords: floating offshore wind turbine, linear frequency domain response, modal analysis.

1. Introduction

Offshore wind resource is much vaster than onshore because wind blows more strongly and consistently offshore. Furthermore, the use of unoccupied marine areas reduces visual and noise annoyance; and turbines are able to have larger dimensions and therefore more power. Floating wind turbines are gaining attention for their ability to capture the wind resource over deep water areas where monopile-supported platforms become uneconomical.

In these floating systems, nacelle oscillating motions have to be minimized in order to limit material fatigue. Coupling between wave induced motion and turbine motion is intended to be significant, and have to be taken into account during design analysis.

Some studies have already been done to evaluate numerically the behaviour of such floating wind turbine systems. Some of theme use linear frequency domain approach to resolve hydrodynamic loads on the platform. Wind turbine influence on the platform is taken into account by increasing loads on the platform with linear aerodynamic damping and linear gyroscopic stiffness [1]-[2].

This study uses the same approach, for the calculation of system movements as in [1]-[2], but with some significant improvements. The effect of the wave direction relative to wind on the system motion is studied. A modal analysis of the system completes this analysis and gives information on system natural motion and on the coupling between system degrees of freedom. This paper is more focused on the description of the methodology than on presentation of the results which are described in more details in [3]

2. Model properties

The system is assumed to undergo rigid body motion. System motions are described in the coordinate system represented on Fig. 1. The coordinate system origin is vertically aligned with the platform gravity center, and is placed in calm surface water line plan. This origin is the reference for system motions, which are described with classical degrees of freedom (DOFs) : surge (1), sway (2), heave (3) for the translations and roll (4), pitch (5), yaw (6) for the rotations.

In this study, wind direction is chosen to be aligned with positive x axis direction. β is the angle between wave direction and wind direction. $\beta = 0$ means that waves are coming from negative x, which means wave direction is aligned with wind direction.

All simulations are realized with an 11.2 m.s^{-1} wind speed. This is the wind speed for which the studied turbine reaches his rated power. For this wind speed the rotor thrust reaches his maximum value.

The wind turbine used in this study is the model known as ≪ NREL offshore 5-MW baseline wind turbine ≫[4]. This concept has been developed at National Renewable Energy Laboratory to serve as a reference of offshore wind development. It does not correspond to an existing turbine, but is a realistic representation of a 5-MW three blade wind turbine. Main properties of this

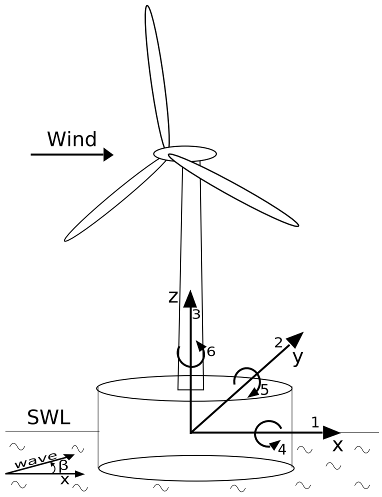


FIGURE 1: Coordinate system and modes of motion

turbine are given in Table 1; all properties are available in [4].

Turbine rate	5 MW
Rotor configuration	3 blades ; upwind
Rotor diameter	126 m
Hub height	90 m
Rotor mass	110 000 kg
Nacelle mass	240 000 kg
Tower mass	347 460 kg
Coordinate of overall CM	(-0.2 m, 0.0 m, 64.0 m)

TABLE 1: Summary of turbine properties

The wind turbine is mounted on the platform known as ≪ MIT/NREL Shallow Drafted Barge (SDB) ≫[1]. This concept has been developed by E. Wayman under the direction of P. Sclavounos, to support the ≪ NREL offshore 5-MW baseline wind turbine ≫. The platform ≪ MIT/NREL SDB ≫is a cylindrical barge. The stability of this design is based on the moment of inertia of the floating surface. The platform is placed in a 200 m water depth.

Design of this barge has been thought to be stable without mooring. Accordingly, the mooring lines provide station keeping functions only. Moorings loads on the platform are taken into account with a stiffness matrix. The only non-zero terms in this matrix are surge-surge and sway-sway stiffness. Table 2 summarizes the properties of the platform, the concept is explained in details in [1] and [2].

Design of ≪ MIT NREL SDB ≫, large floating surface, is in the domain of validity of the hydrodynamic modelling of this study. In fact, for linear frequency domain hydrodynamic theory, floater dimensions have to be larger than wave amplitude.

Diameter	36 m
Draft, Freeboard	5 m, 4.5 m
Mass, ballast height	$4153 \cdot 10^3$ kg, 1.595 m
Total mass	$4519 \cdot 10^3$ kg
CM location	-3.9 m
Roll, pitch inertia about CM	$3.9 \cdot 10^8$ kg.m^{-2}
Yaw inertia about CM	$7.5 \cdot 10^8$ kg.m^{-2}

TABLE 2: Summary of platform properties

3. Methodology

3.1 Overall study approach

This study is composed of two parts, a dynamic analysis and a modal analysis. The dynamic analysis part resolves the Response Amplitude Operators (RAOs) of the floating wind turbine system under wave loads. This dynamic analysis is based on the methodology described in [2], but takes into account significant differences which are explained further (especially for the wind turbine linearization). The modal analysis part completes the dynamic analysis. It provides access to natural frequencies, natural modes and modal damping coefficients of the floating wind turbine system. These information help to understand the forced motion of the structure and highlight the coupling between the DOFs of the system. Links between the different steps of this study are represented in Fig. 2.

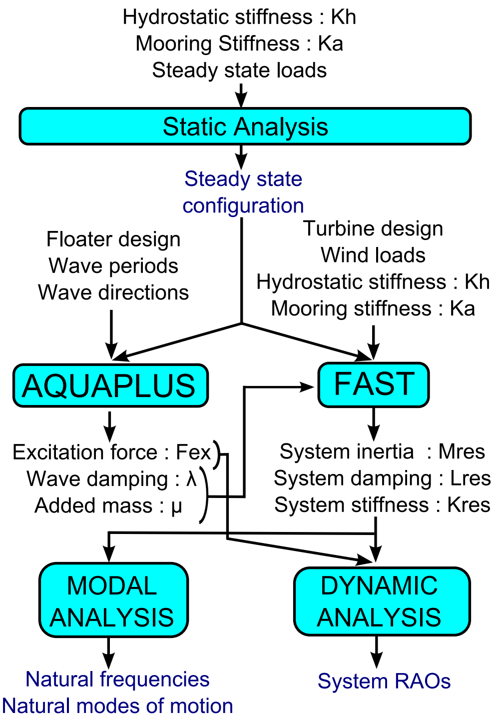


FIGURE 2: Overall study approach

The total external load on the system: platform, mooring, wind turbine comes from wind and waves. In this study we consider the system as a rigid body, and we calculate the motions of the platform. Loads acting on the platform are: fluid reaction F_{hydro}, mooring loads F_a and loads transmitted from the turbine to the platform F_{aero}. System equation of motions is :

$$M_p \ddot{q} = F_{aero} + F_{ancrage} + F_{hydro} \qquad (1)$$

with M_p the platform inertia matrix, and q the vector of degrees of freedom.

The first step of the study is to find the steady state operating point of the system, around which it will oscillate due to periodic wave loads.

Hydrodynamic wave loads on the platform are calculated with Aquaplus. Aquaplus is a linear frequency domain diffraction/radiation code developed at Laboratoire de Mécanique des Fluides from Ecole Centrale Nantes [5]. This codes calculates for each wave frequency ω, the radiation force in terms of added mass $\mu(\omega)$ and wave damping $\lambda(\omega)$, and the wave induced exciting force $F_{ex}(\omega, \beta)$ (including diffraction loads) [6]. This calculation is done considering the platform at the operating point found with the static analysis. This is an improvement regarding [1], [2], who calculates, according with linear theory, hydrodynamic properties in the undisturbed position. Symmetry of the system is perturbed. It introduces coupling between system motions in the calculation of hydrodynamic properties.

Mooring loads on the system are modelled with a mooring stiffness matrix K_a. Hydrostatic forces are taken into account with a hydrostatic stiffness matrix K_h.

FAST, an aero-elastic simulation code for wind turbine developed at NREL, has the ability to linearize a model about an operating point [7]. The model is linearized about the steady state operating point found in the static analysis. The model built in FAST takes into account mooring loads, hydrostatic forces and hydrodynamic radiation forces calculated with Aquaplus. The model used in FAST linearization is explained in details in section 3.3 .

FAST outputs the resulting mass, damping and stiffness matrices M_{res}, L_{res}, K_{res} of the whole system: platform, wind turbine, mooring system. Finally, motion equation of the system under periodic wave loads in frequency domain is with complex notation:

$$\left(-\omega^2 M_{res}(\omega) + i\omega L_{res}(\omega) + K_{res}(\omega)\right) \Delta q = F_{ex} \qquad (2)$$

with M_{res}, L_{res}, K_{res} depending on incident wave frequency ω. Δq is the vector of system displacement around steady state position. The different components from resulting matrices M_{res}, L_{res}, K_{res} are detailed in section 3.3 . Resolution of equation 2 for a 1 meter incident wave gives access to system motion RAOs under wave loads

The second part of this study is a modal analysis. It consists in the calculation of natural frequencies and modes and modal damping coefficients of the whole system: platform, turbine, mooring, by resolving equation (2) for free motion (without forcing term F_{ex}). This calculation is explained in details in section 3.4.

3.2 Static analysis

Steady state position of the system is found by resolving equation (3) :

$$F_{aeros} + F_a + W + F_{hydrostatic} = 0 \qquad (3)$$

with F_a is mooring loads, W weight of the system, $F_{hydrostatic}$ hydrostatic force on the platform and F_{aeros} steady state wind loads. This steady state wind loads takes into account contribution of rotor thrust and rotor torque. These steady state forces are given in [4], and can be recalculated with FAST. As we model F_a with a stiffness matrix, equation (3) becomes :

$$(K_h + K_a) X_S = F_{aeros} \qquad (4)$$

with X_S the steady state position. F_{aeros} expression is $F_{aeros} = (F_{thrust},\ 0,\ 0,\ C_{torque},\ F_{thrust} \cdot z_{hub},\ 0)^t$. In this study there is no yaw and excitation stiffness. Consequently, it is considered that there is no steady state yaw displacement.

3.3 FAST Linearization

FAST is able to compute a linearized representation of a non linear wind turbine model. The procedure is explained in details in [7]. FAST also brings the possibility to define user loads on the support platform. In this section we detail the linearization process in the case of our user defined platform loads model.

In FAST, the complete non linear equations of motions express themselves as

$$M_F\left(q, u, t\right) \ddot{q} + f_F\left(q, \dot{q}, u, u_d, t\right) = 0 \qquad (5)$$

with M_F the mass matrix, f_F the non linear forcing vector, q the DOFs vector, u the vector of control inputs, u_d the vector of wind input disturbance and t the time.

In this case we consider no wind disturbance and no control input. The equation solved by FAST, in case of a floating wind turbine moored on the seabed and without incident wave is

$$\begin{aligned}\left[M_e(q,t) + M_p(q,t)\right] \ddot{q} + f(q, \dot{q}, t) + F_{rad}(q, \dot{q}, t) \\ + F_b(q,t) + F_a(q,t) = 0 \qquad (6)\end{aligned}$$

with M_e the turbine mass matrix and M_p the platform one. f is the forcing function from external loads on the system excepting :

- F_a, contribution from mooring loads
- F_b, contribution from buoyancy
- F_{rad}, contribution from radiation force previously calculated with Aquaplus.

FAST numerically linearizes this equation by perturbing each value from his value at operating point. The value at operating point is noted with suffix $_{op}$:

$$q = q_{op} + \Delta q \ \ ; \ \ \dot{q} = \dot{q}_{op} + \Delta \dot{q} \ \ ; \ \ \ddot{q} = \ddot{q}_{op} + \Delta \ddot{q} \quad (7)$$

System oscillating motion around the operating point can be write by linearizing Eq. (6) :

$$\underbrace{\left(\underbrace{M_e|_{op}+M_p|_{op}}_{(1)}+\underbrace{\left.\frac{\partial F_{rad}}{\partial \ddot{q}}\right|_{op}}_{(2)}\right)}_{M_{res}}\Delta\ddot{q}(t) + \underbrace{\left(\underbrace{\left.\frac{\partial f}{\partial \dot{q}}\right|_{op}}_{(3)}+\underbrace{\left.\frac{\partial F_{rad}}{\partial \dot{q}}\right|_{op}}_{(4)}\right)}_{L_{res}}\Delta\dot{q}(t) + \underbrace{\left(\underbrace{\left.\frac{\partial f}{\partial q}\right|_{op}+\left.\frac{\partial M_e+M_p}{\partial q}\right|_{op}\ddot{q}}_{(5)}+\underbrace{\left.\frac{\partial F_b}{\partial q}\right|_{op}}_{(6)}+\underbrace{\left.\frac{\partial F_a}{\partial q}\right|_{op}}_{(7)}\right)}_{K_{res}}\Delta q(t)=0 \quad (8)$$

In Eq. (8), term (1) is the mass matrix of the system at operating point. Term (2) is the added mass matrix. Term (3) is the aerodynamic damping. Term (4) is the wave damping. Term (5) is the added stiffness which includes gyroscopic stiffness. Term (6) is the hydrostatic stiffness and term (7) is the mooring stiffness. FAST gives access to the resulting matrix M_{res}, L_{res}, and K_{res}.

Equation of oscillating motion of the system around operating point due to an harmonic excitation of incident wave is :

$$M_{res}\Delta\ddot{q}(t)+L_{res}\Delta\dot{q}(t)+K_{res}\Delta q(t)=F_{ex}(t) \quad (9)$$

Using complex notation $\Delta q(t) = \Delta q e^{i\omega t}$, Eq. (9) becomes :

$$\left(-\omega^2 M_{res}(\omega)+i\omega L_{res}(\omega)+K_{res}(\omega)\right)\Delta q = F_{ex} \quad (10)$$

with F_{ex} wave excitation force (including diffraction) due to an incident wave at frequency ω. In Eq. (10), the ω dependency of M_{res} comes from the added mass, the one from L_{res} comes from wave damping, and the one from K_{res} comes from the acceleration dependency of this matrix.

In their study Wayman et al. [1] neglected the mooring, hydrostatic and radiation loads for FAST linearization. As a consequence they obtained as FAST output only the contribution from wind turbine on the system. They add this contribution to motion of equation of the platform to get motion equation of the whole system. The methodology used in this study permits to get better results because the acceleration term in term (5) from Eq. (8) is calculated by taking into account the whole loads on the system. Consequently Wayman et al. obtained representatives matrices for the turbine which do not depend from ω, contrarily to the present study.

3.4 Modal Analysis

Free oscillation of the system around steady state position is Eq. (10) without forcing vector:

$$M_{res}\Delta\ddot{q}(t)+L_{res}\Delta\dot{q}(t)+K_{res}\Delta q(t)=0 \quad (11)$$

In Eq. (11), L_{res} and K_{res} are not symmetric due to rotor rotation which leads to asymmetric aerodynamic damping and gyroscopic stiffness. Modal resolution has been done following the methodology for damped rotating system presented in [8]. This methodology is summarized further.

Furthermore matrices M_{res}, L_{res}, K_{res} depend on ω. To obtain accurate results, modal resolution has been done for each natural frequency by considering M_{res}, L_{res}, K_{res} at their value at natural frequency. The natural frequencies are found by iteration. Resolution of Eq. (11) at ω_i brings natural mode i

For each mode of motion, the solution of Eq. (11) is found with the classical exponential form Eq. (12):

$$\Delta q(t)=\gamma p e^{\lambda t} \quad (12)$$

with γ, λ, p, complex values describing respectively a reference of amplitude, a scalar coefficient, and a vector of constant. It transforms the resolution in a quadratic eigenvalue problem Eq. (13), where λ and p are the corresponding eigenvalue and eigenvector :

$$(\lambda^2 M+\lambda L+K)p=0 \quad (13)$$

The following substitution [8], transforms the linear 6x6 eigenvalue problem in a linear 12x12 eigenvalue problem.

$$g = \{q,\dot{q}\}^T \quad (14)$$

$$A = \begin{pmatrix} 0 & K \\ K & L \end{pmatrix} \quad (15)$$

$$B = \begin{pmatrix} K & 0 \\ 0 & -M \end{pmatrix} \quad (16)$$

The system to solve becomes

$$B\dot{g}-Ag=0 \quad (17)$$

and the associated linear eigenvalue problem is Eq. (18), with associated eigenvector u is $u=\{p,\lambda p\}^T$. λ and p are the eigenvalue and eigenvector of the initial quadratic problem Eq. (13).

$$(A-\lambda B)\,u=0 \quad (18)$$

Eigenvalues appear as n pairs of complex conjugate values $\lambda_i=-\alpha_i+j\omega_i$ and $\bar{\lambda_i}=-\alpha_i-j\omega_i$. α_i and ω_i are respectively the natural damping coefficients and the natural frequencies of the system. Eigenvectors appear as n pairs of conjugate vector u_i and $\bar{u_i}$ of size 2n. First n components are the modal vector p_i et $\bar{p_i}$ of the structure.

We note ρ_i the vector of modulus of p_i and $\bar{p_i}$, and ϕ_i vector of phases of p_i. The researched solution expresses as follow :

$$q_{ij}(t) = \gamma \rho_i j e^{-\alpha_i t} e^{i(\omega_i t + \phi_{ij})} \quad (19)$$

$$\overline{q_{ij}}(t) = \gamma \rho_i j e^{-\alpha_i t} e^{i(\omega_i t - \phi_{ij})} \quad (20)$$

Vectors ρ_i are the amplitude of natural modes at natural frequencies ω_i, with damping coefficient α_i. Vectors ϕ_i represent the phases between motions of mode i.

4. Results

4.1 Steady state position

Steady state properties of the studied system are given in Table 3. Steady state pitch is lower than 10˚, which is the supposed limit position from which the turbine will loose significant efficiency.

$K_h(3,3)$	$1.0 \cdot 10^7$ Nm^{-1}
$K_h(4,4)$, $K_h(5,5)$	$4.4 \cdot 10^8$ Nm
Steady state pitch	9.6˚
Steady state roll	0.5˚

TABLE 3: Steady state position of the platform

4.2 Natural frequencies and modes of motion

Natural frequencies of the system are given (in rad.s^{-1}) in the vector Ω. We call mode 1 to 6 the natural modes corresponding to natural frequencies in the order they are ranged in vector Ω. Matrix ρ columns are the natural modes 1 to 6 associated with the corresponding 1 to 6 Natural frequencies. Vector α represents the modal damping coefficient associated with these modes. Matrix Φ represents the phases associated with the natural modes of motions

$$\Omega = \begin{pmatrix} 0.9 \\ 0.7 \\ 0.8 \\ 0.4 \\ 0.3 \\ 0.4 \end{pmatrix} \quad (21)$$

$$\rho = \begin{pmatrix} 0.4 & 0.8 & 0.0 & 0.0 & 0.7 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.7 & 0.1 & 0.7 \\ 0.6 & 0.2 & 0.0 & 0.0 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.4 & 22.8 & 3.3 & 21.8 \\ 0.3 & 0.5 & 0.0 & 1.5 & 32.0 & 1.4 \\ 0.0 & 0.0 & 0.1 & 28.9 & 0.7 & 29.3 \end{pmatrix} \quad (22)$$

$$\alpha = \begin{pmatrix} 0.1 \\ 0.0 \\ 0.0 \\ -0.2 \\ 0.0 \\ 0.3 \end{pmatrix} \quad (23)$$

$$\Phi = \begin{pmatrix} 178.2 & 0.0 & -3.7 & -17.3 & 180.0 & -15.3 \\ -7.5 & -167.0 & 0.0 & 0.0 & 56.6 & 0.0 \\ 0.0 & -18.1 & -117.7 & -40.3 & -173.5 & -144.3 \\ -11.9 & -168.6 & -11.8 & 158.1 & -125.2 & -161.5 \\ -10.4 & 167.2 & 164.4 & 13.5 & -179.5 & 64.6 \\ -117.8 & 76.1 & -13.2 & -125.3 & -164.2 & 129.0 \end{pmatrix} \quad (24)$$

The six natural frequencies of this system are located in range of values that are excited by ocean wave spectrum. The first mode of motion at 0.9 rad.s^{-1} is dominated by heave and is coupled with pitch and surge. The second mode at 0.7 rad.s^{-1} is similar with the same coupled motions but is dominated by surge. These two modes show a coupling between the surge, pitch and heave motions of the system. The third mode at 0.8 rad/s is dominated by sway, and shows a important coupling with roll and a smaller with yaw. Natural frequencies 1, 2 and 3 are close. The corresponding modes can be excited together. The fifth mode is dominated by pitch, and shows a small coupling with the other five DOFs. Coupling with translation surge, sway, heave is negligible, but the pitch-roll coupling is observable with a value of 10 %. Modal damping coefficients of these modes 1, 2, 3 and 5 are positive, these natural modes are damped.

Natural frequencies 4 and 5 are at the same value of 0.4 rad.s^{-1}. The corresponding modal amplitude ρ are approximately the same, but phases between motions are different. Modal damping coefficients of mode 4 is negative valued, sign of a possible loss of stability [8]. These modes show a very important coupling between roll and yaw motion. This coupling is not at the same phase in the two motions. Consequently, the system oscillates inversely in the two modes. These two modes are at the same frequency and cancel each other.

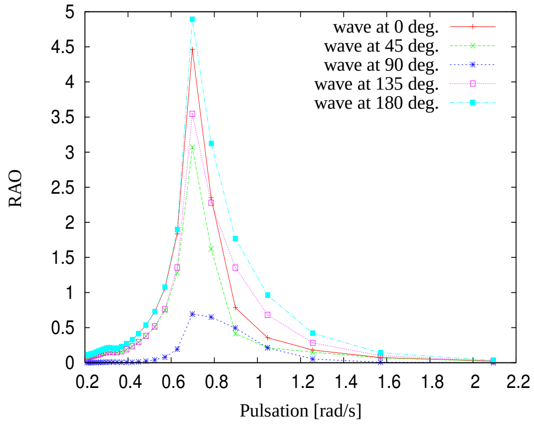
4.3 System RAOs regarding wave direction

Fig. 3 represents the displacement RAOs of the system: platform, mooring, wind turbine for 5 wave directions from 0˚ to 180˚. 0˚ means wave direction is aligned with wind direction.

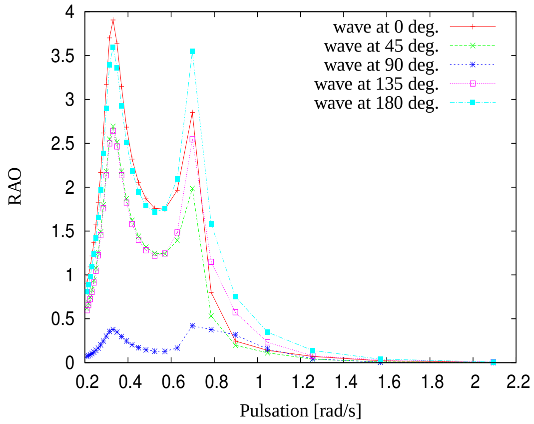
Surge RAO, represented on Fig. 3a, shows a large peak around 0.7 rad/s for all wave direction. When wave is aligned with x axis, mode 2, dominated by surge, is excited, this explain the important response for these wave directions. For wave coming from 90˚, surge motion is not zero. Due to the steady state position, the floater is non symmetric, so the surge wave excitation/diffraction force is non-zero for 90˚ wave direction. Furthermore, for 90˚ at 0.7 rad/s wave, heave component of mode 1 and 2 can be excited, and the system moves in surge due to the coupling between surge and heave for these modes.

The pitch response is represented on Fig. 3b. Motions are represented around the steady state position, which is 9.6˚ in pitch. The curves show two peaks around 0.3 rad.s^{-1} and around 0.7 rad.s^{-1}. The first one around 0.3 rad.s^{-1} comes from excitation of mode 5 dominated by pitch. The second around 0.7 rad.s^{-1} comes from excitation of modes 1 and mode 2 which contain a pitch component. When waves come from 90˚, pitch motion is small, and not easy to explain. It may come from excitation of roll-pitch coupling in mode 4 and 6. Motion amplitudes are small because these modes cancel one with each other.

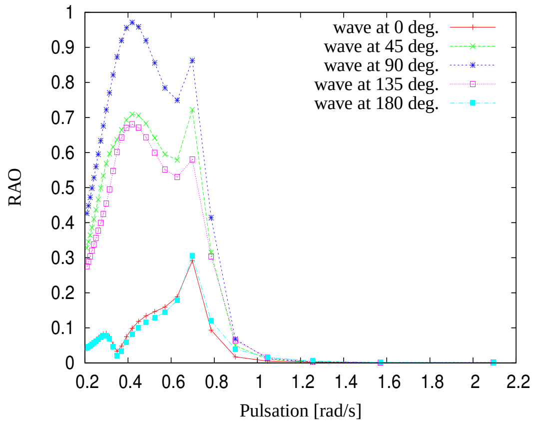
The yaw response on Fig. 3c is maximal when wind direction crosses wind direction. It's interesting to notice that the yaw motion comes from the coupling, because there is no yaw excitation. There is yaw coupling in the Natural modes 3, 4, 5 and 6. Strong coupling between roll and yaw in modes 4 and 6 may explain the more important motion when wave direction crosses wind.



(a) surge RAO



(b) pitch RAO



(c) yaw RAO

FIGURE 3: System RAOs regarding wave direction

5. Conclusions

This study highlights importance of modal analysis to better understand the behaviour of floating wind turbine. It brings out natural frequencies around which system motions are important, and natural modes which help to understand the coupling between the system DOFs. Effect of wave direction relative to wind has been studied for a particular floating wind turbine system. It appears that natural modes are excited differently regarding wave direction. The case where wind and wave are not aligned brings out maximal yaw response.

In this study the wind turbine is modelled with damping and stiffness matrices. Others have developed time domain simulation tools which give access to model with more DOFs for the wind turbine [9]. It should be interesting to study the effect of wave direction relative to wind in time domain for transient events.

Furthermore, to be able to predict correctly the behaviour of floating wind turbine system, it will be necessary to study the effect of non linear hydrodynamic on the coupling between system motions.

Acknowledgements

The authors would like to acknowledge ADEME (the French environment agency) and Région Pays de la Loire for funding the Phd program in which this study has been done.

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